Controller Bandwidth Candidate

Background theory from "Modern Control Engineering Fifth Edition"

Cutoff Frequency and Bandwidth. Referring to Figure 7–76, the frequency ω_b at which the magnitude of the closed-loop frequency response is 3 dB below its zero-frequency value is called the *cutoff frequency*. Thus

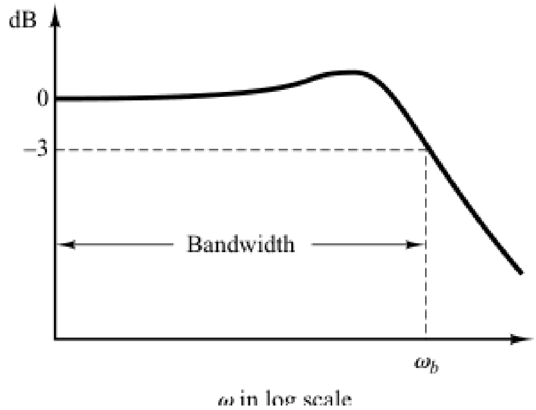
$$\left|\frac{C(j\omega)}{R(j\omega)}\right| < \left|\frac{C(j0)}{R(j0)}\right| - 3 \text{ dB}, \qquad \text{for } \omega > \omega_b$$

For systems in which $|C(j0)/R(j0)| = 0$ dB,

$$\left|\frac{C(j\omega)}{R(j\omega)}\right| < -3 \text{ dB}, \qquad \text{for } \omega > \omega_b$$

The closed-loop system filters out the signal components whose frequencies are greater than the cutoff frequency and transmits those signal components with frequencies lower than the cutoff frequency.

The frequency range $0 \leq \omega \leq \omega_b$ in which the magnitude of $C(j\omega)/R(j\omega)$ is greater than -3 dB is called the *bandwidth* of the system. The bandwidth indicates the frequency where the gain starts to fall off from its low-frequency value. Thus, the bandwidth indicates how well the system will track an input sinusoid. Note that for a given ω_n, the rise time increases with increasing damping ratio ζ. On the other hand, the bandwidth decreases with the increase in ζ. Therefore, the rise time and the bandwidth are inversely proportional to each other.



The specification of the bandwidth may be determined by the following factors:

1. The ability to reproduce the input signal. A large bandwidth corresponds to a small rise time, or fast response. Roughly speaking, we can say that the bandwidth is proportional to the speed of response. (For example, to decrease the rise time in the step response by a factor of 2, the bandwidth must be increased by approximately a factor of 2.)
2. The necessary filtering characteristics for high-frequency noise.

For the system to follow arbitrary inputs accurately, it must have a large bandwidth. From the viewpoint of noise, however, the bandwidth should not be too large. Thus, there are conflicting requirements on the bandwidth, and a compromise is usually necessary for good design. Note that a system with large bandwidth requires high-performance components, so the cost of components usually increases with the bandwidth.

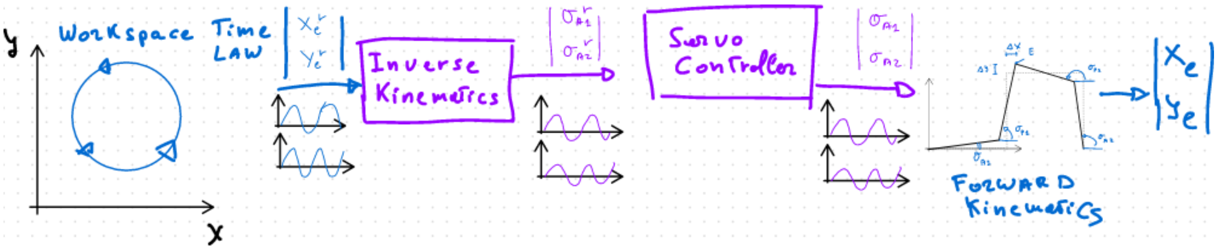
1) The basic Idea

My proposal is to focus the Bandwidth ω_b specification on avoiding the non-linearities.

We know:

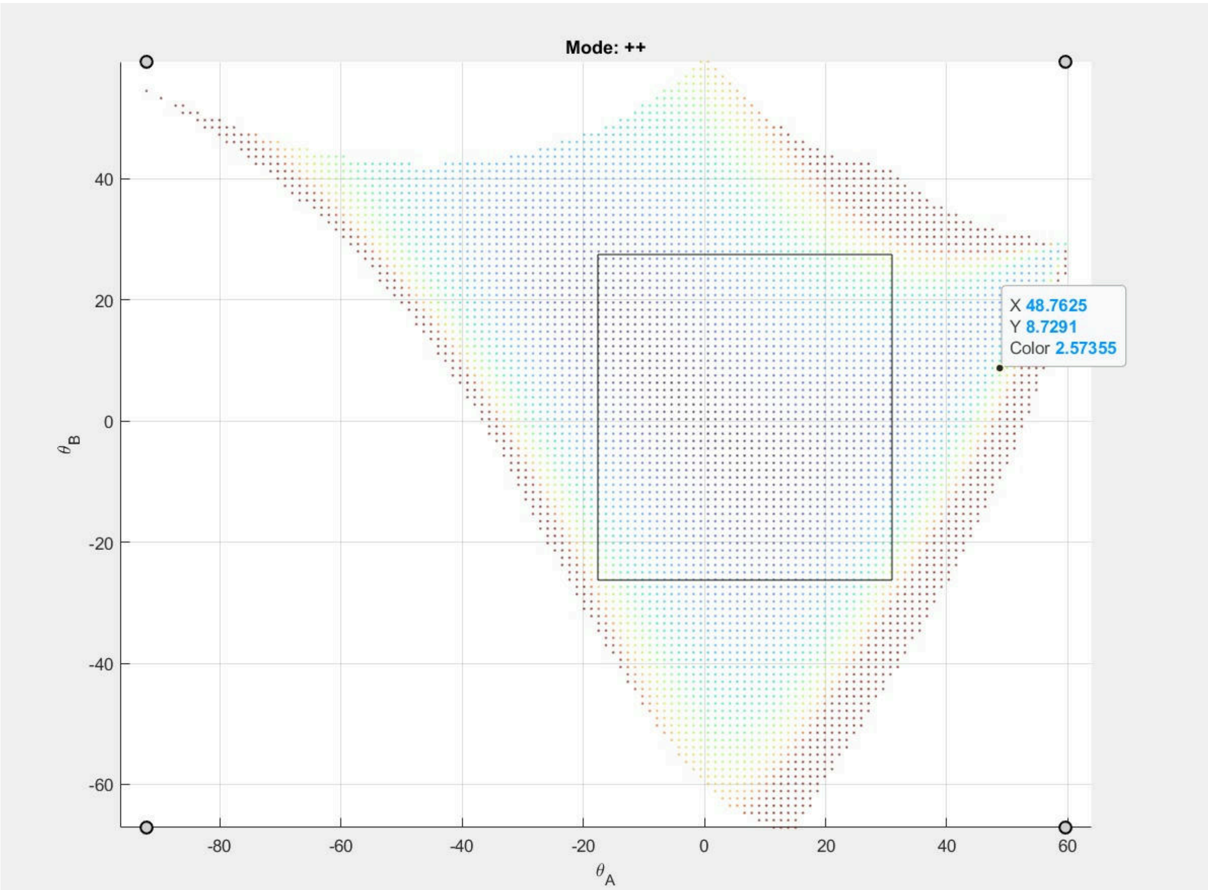
- The available joint space in radians.
- The voltage→angle transfer function $G_\theta(j\omega)$.
- The voltage limits of the power supply.

Suppose that we want to control the angular position with a sinusoidal reference bounded between θ_{max} and θ_{min}. Ideally this signal would come from an Inverse Kinematics module upstream, which in turns is trying to follow a **continuos** shape in the cartesian workspace.



Let's assume for now that the Inverse Kinematics module doesn't add higher order frequencies to the output signals.

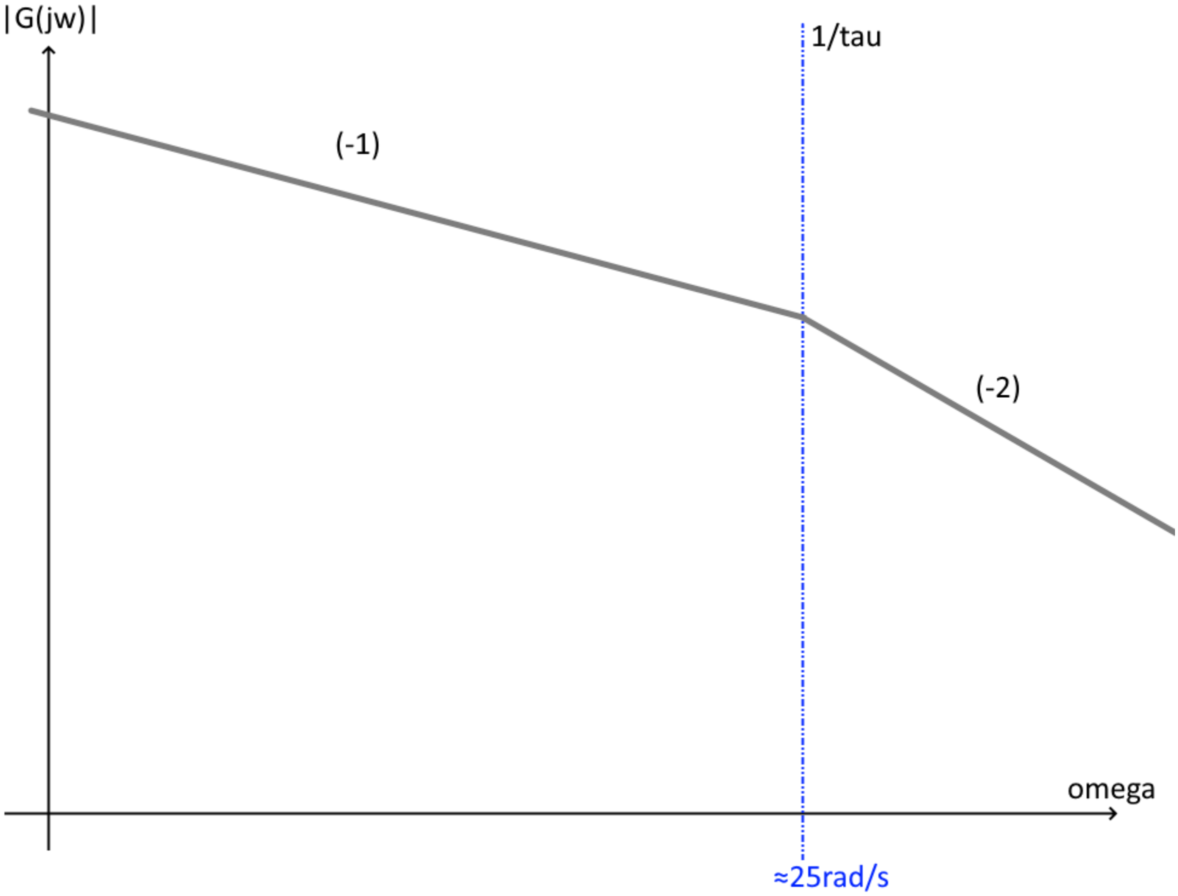
Let's assume that we want the Inverse Kinematics module to use the whole **"allowed joint space rectangle"** (AJSR from now on) and that we want the servo controller to be able to track whichever signal up to ω_b.



From all the analysis we know that the angular transfer function is

$$G_\theta(j\omega) = \frac{1}{s}\frac{\mu}{s\tau + 1} \tag{1.}$$

The bode plot of the gain is:



We can interpret this as: Given a voltage with amplitude A_v and frequency $\overline{\omega}$, the shaft angle at steady state will oscillate with the same frequency and with an amplitude

$$A_\theta = A_v\ |G_\theta(j\overline{\omega})| \tag{2.}$$

We know that we have a limit on the voltage, so we are interested to know at which frequency the gain $|G_\theta(j\omega)|$ is not enough to cover the whole AJSR amplitude. From the graph is easy to see how this happens at higher frequency.

2) Implementation

Let's focus on Motor 0, but this argument can extended on Motor 1 without loss of generality.

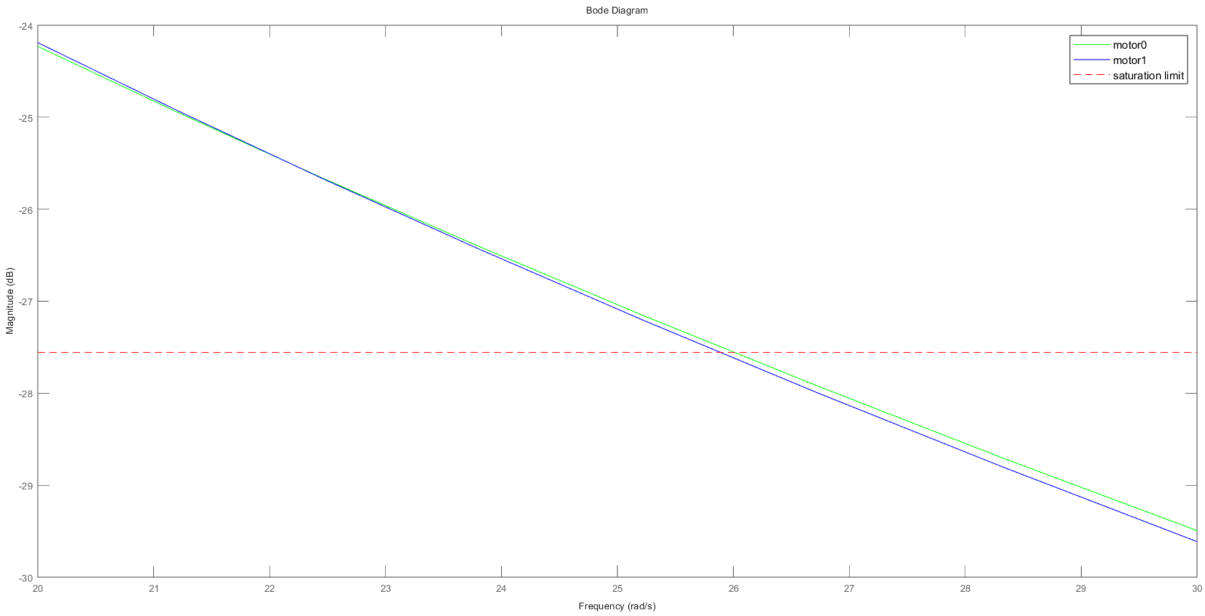
Motor0 can move from $-17°$ to $+31°$, so a sinusoidal angular movement should cover in total $\theta_{pp} = 48° = 0.83$ rad

Both motors can be driven up to ±10V for "short periods" of time which translates in $V_{pp} = 20V$

The minimum gain (at the maximum frequency) is

$$|G_\theta^{\min}(j\omega_{\max})| = \frac{0.83}{20} = 0.0419 = -27.5582\ \text{dB} \tag{3.}$$

Let's plot the bode gain and see where it crosses -27dB:



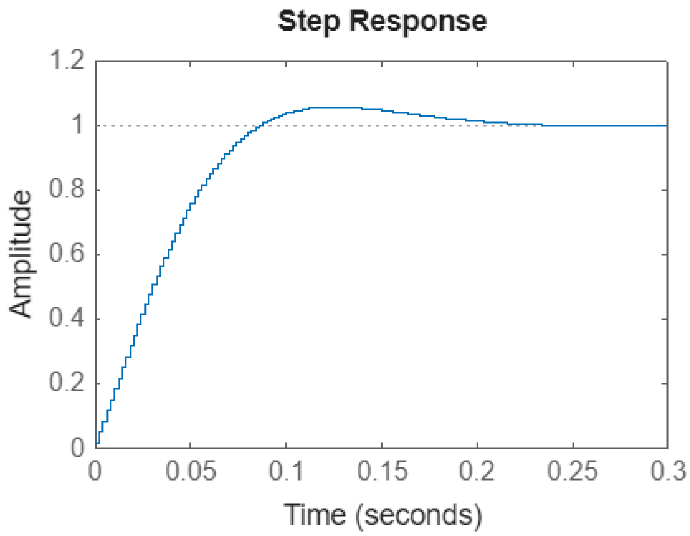
It happens at around **26rad/s.**

3) Controller synthesys and verification

I designed a simple PD controller with MATLAB's PIDTUNE():

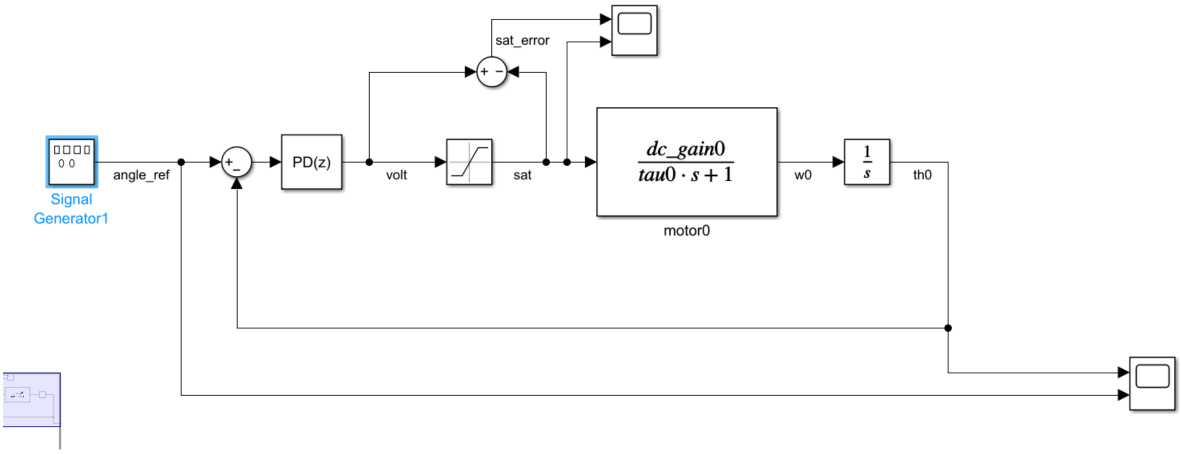
```
C_pd0 =

              z-1
  Kp + Kd * ------
               Ts

  with Kp = -21.4, Kd = -0.43, Ts = 0.002

Sample time: 0.002 seconds
Discrete-time PD controller in parallel form.
Model Properties
```

```
info = struct with fields:
                Stable: 1
      CrossoverFrequency: 26
           PhaseMargin: 70.0000
```

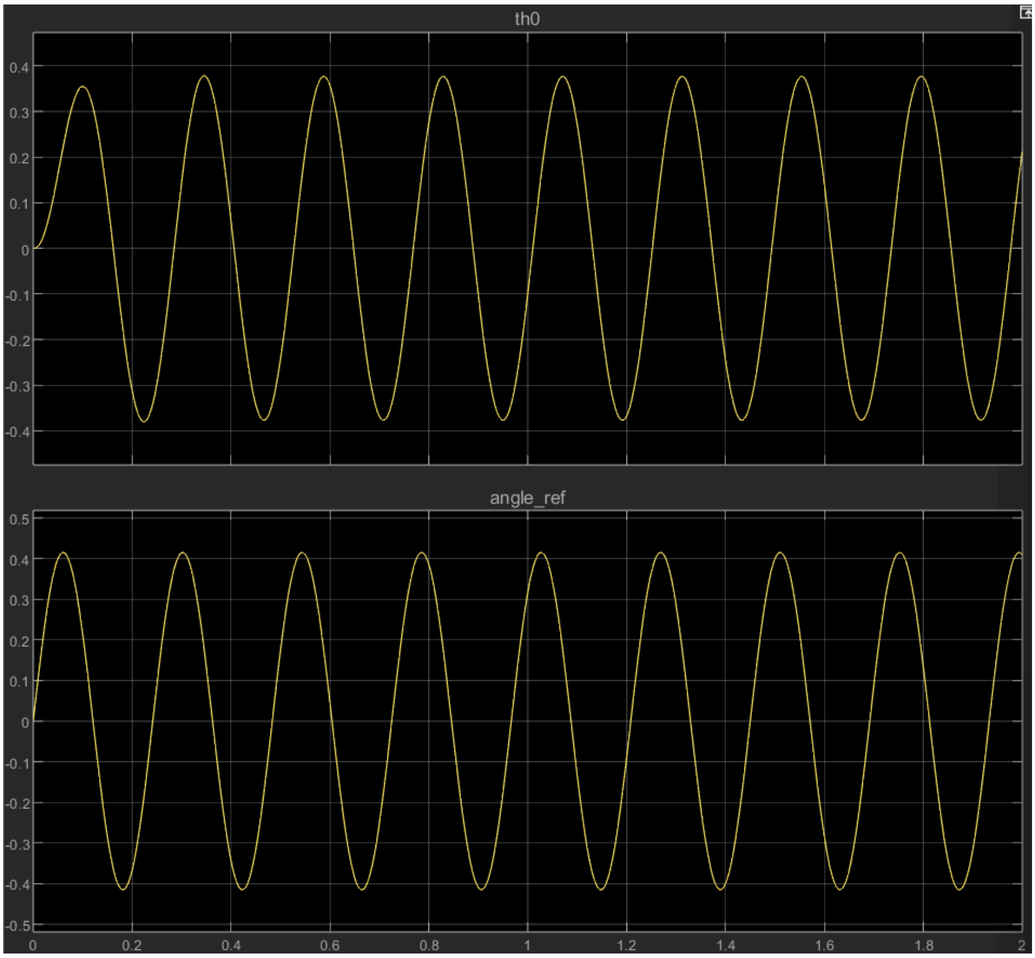


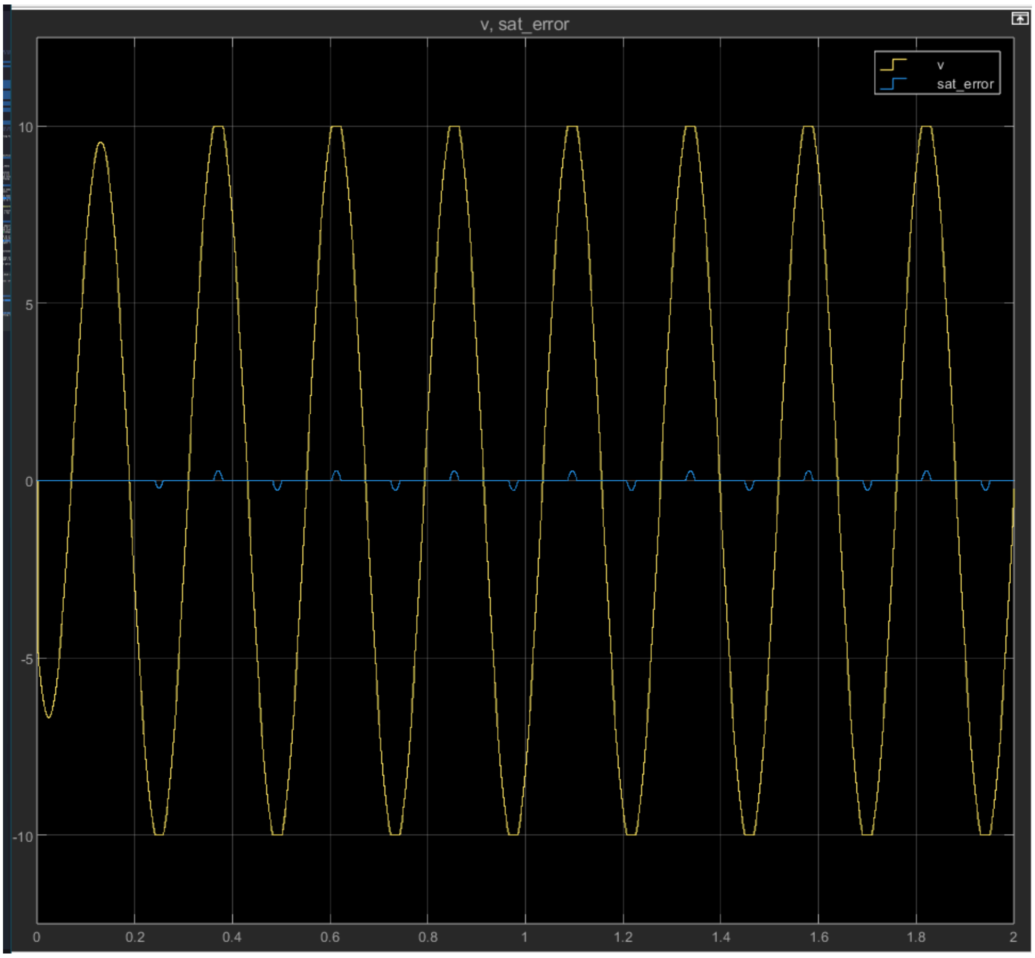
$$\begin{cases} \text{Kp} = -21.4 \\ \text{Kd} = -0.43 \end{cases} \tag{4.}$$

Then with a simple SIMULINK scheme I tested if it was able to follow a sinusoidal angle reference without hitting the saturation limits:



Test at 26 rad/s, amplitude $= \frac{\theta_{\text{pp}}}{2} = 0.4189$ rad





It almost follows the angle trajectory in its full range with some saturation (see blue line in the second graph).

This is due to the fact that at 26rad/s we are already at -3dB (70%) in terms gain.

4) Comments

4.1) It feels a little bit too slow

The group of the previous year used 30rad/s as the tuning bandwidth if I recall correctly, this is not too slow if compared to them and it's from a "rigorous" analysis.

4.2) 70° of phase margin comes from?

It's the standard phase margin of the automation course. Too high means increased controller effort (more voltage, more saturation), too low and the system oscillates in case of step response (we may overshoot the AJSR).

We can increase the phase margin but at the cost of bandwidth (design decision, let's what happens in the lab).

4.3) What to do in the lab

Implement PD controller, check if we have saturations when trying with the AJSR at 26rad/s.