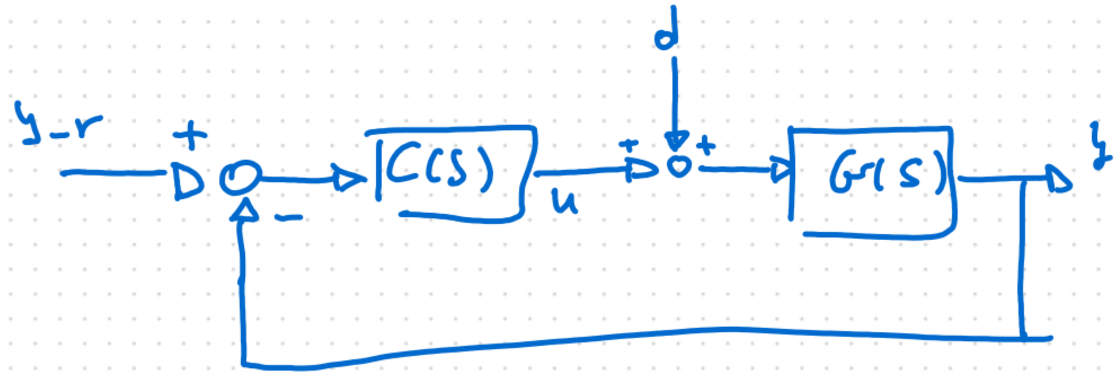
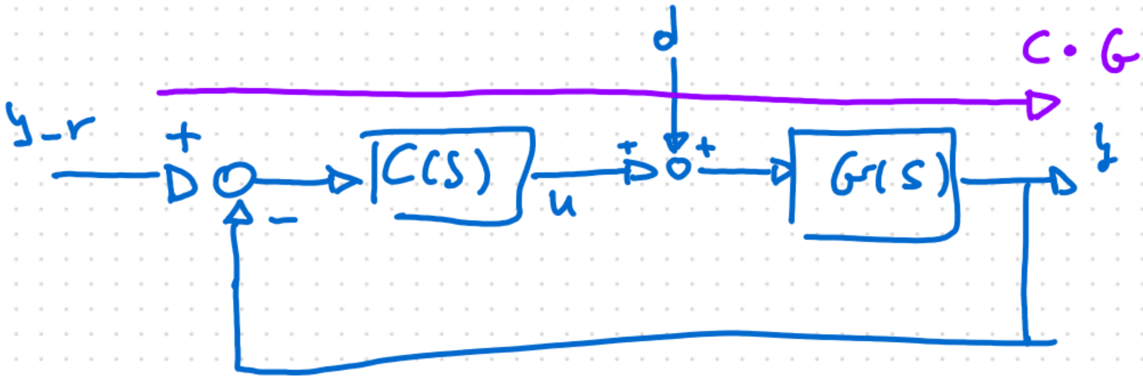
Sensitivity

This is the scheme we used in LAB4:



We ran mainly two types of experiments: reference following and disturbance rejection.

1) Reference following



We are seeing "how good is our control loop at following a reference signal". This is described in frequency by the ***COMPLEMENTARY SENSITIVITY TRANSFER FUNCTION T(S)***:

$$C(s)^{\text{PD}}_{\text{ideal}} = P + Ds = P\left(s\frac{P}{D} + 1\right) = P(sT + 1) \quad 1.$$

With $T = \frac{P}{D}$ = time constant of our controller.

$$G(s) = \frac{\mu}{s(\tau s + 1)} \quad 2.$$

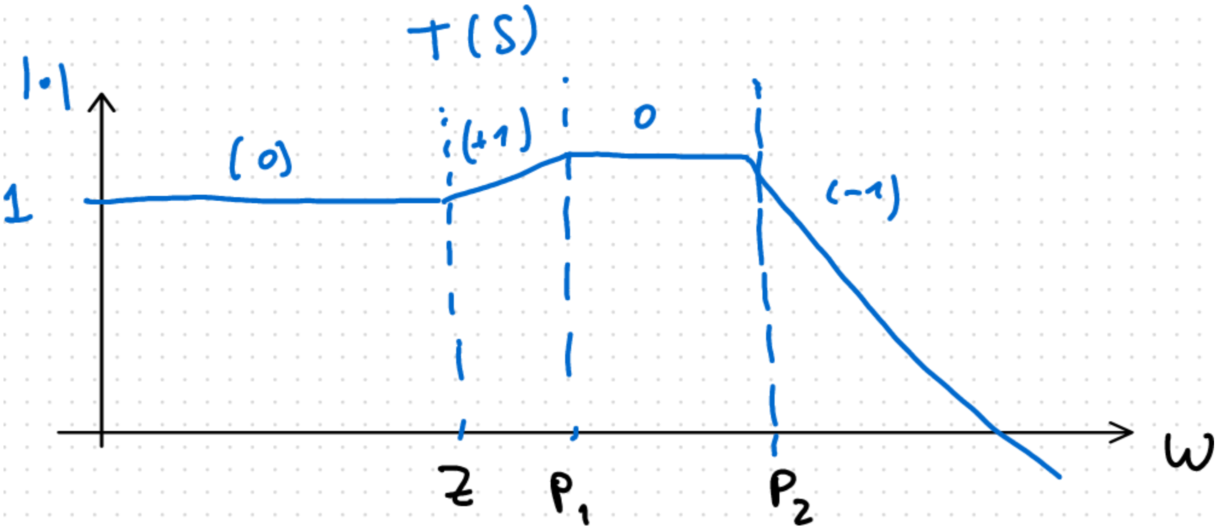
$$L(s) = C(s)G(s) = \frac{P\mu(sT + 1)}{s(\tau s + 1)} = \frac{N(s)}{D(s)} \quad 3.$$

$$T(s) = \frac{L(s)}{1+L(s)} = \frac{\frac{N(s)}{D(s)}}{1+\frac{N(s)}{D(s)}} = \frac{N(s)}{D(s)+N(s)} = \tag*{4.}$$
$$= \frac{P\mu(sT+1)}{s(\tau s+1)+P\mu(sT+1)} = \frac{P\mu(sT+1)}{s^2\tau + s(P\mu T+1)+P\mu}$$

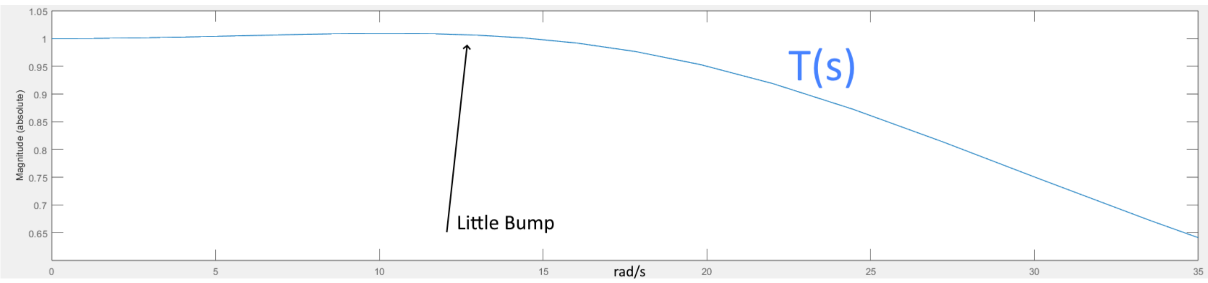
We expect two poles (P1&P2) and one zero (Z).

The static gain is: $T(0) = \frac{P\mu}{P\mu} = 1$ (Thanks to the integrator from the motor!)

The asymptotic graph has the following shape:



In MATLAB, using the plant and controller we get:



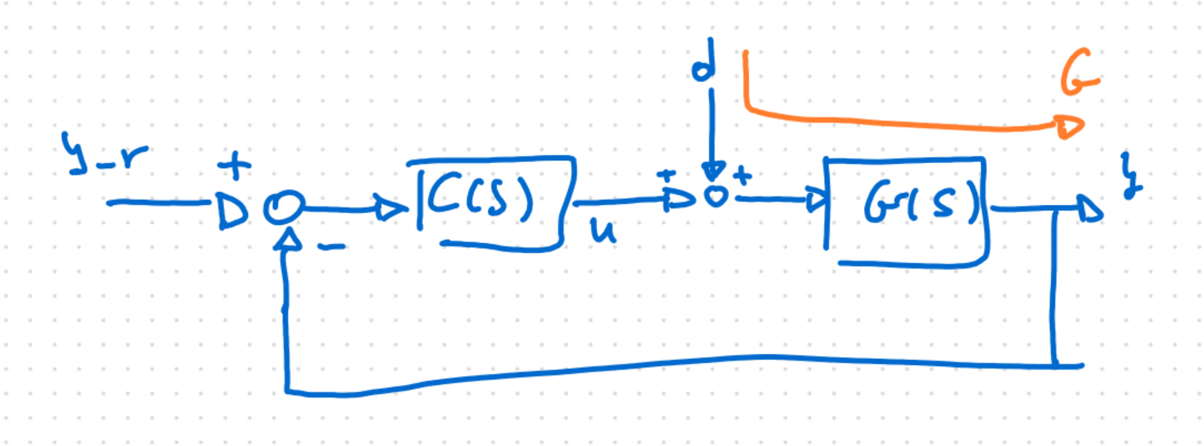
(There is a bump, trust me)

If you remember the reference tracking experiment we had a good tracking at "low" frequency, some overshoot at medium and undershoot at high.

TODO:

- Analyze the frequency response from the experiment
- It matches in shape, but will it also overlap in value???

2) Input Sensitivity Function

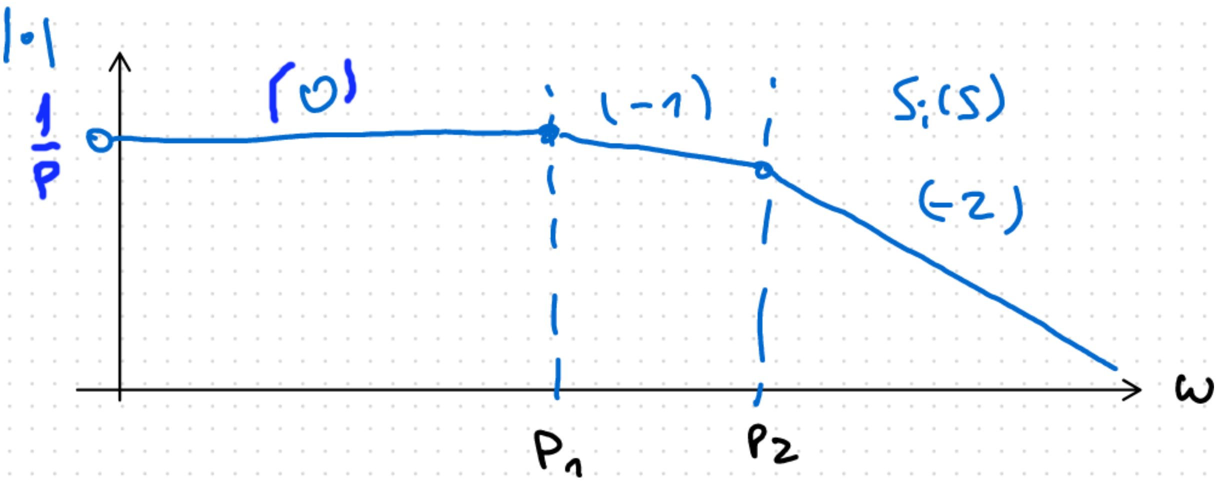


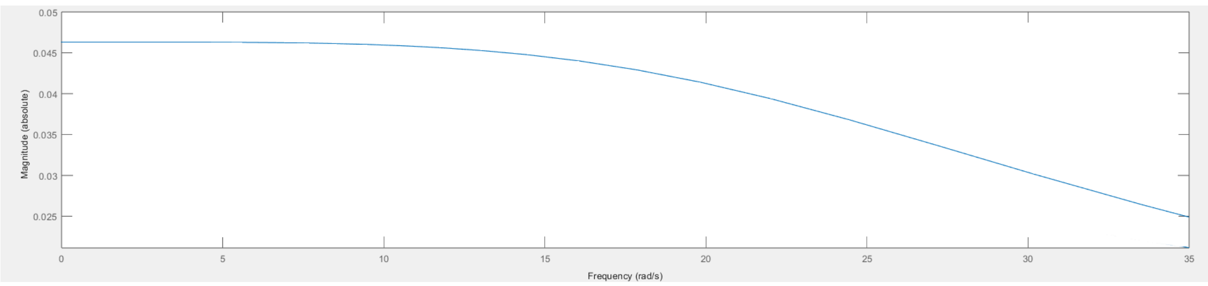
Sometimes called **LOAD DISTURBANCE SENSITIVITY FUNCTION**, it describes how good is our system at dealing with disturbance in the load. It's the only "replicable" experiment that we can do so we have to do it.

$$S_i(s) = \frac{G(s)}{1+L(s)} = \frac{\frac{\mu}{s(\tau s+1)}}{1+\frac{P\mu(sT+1)}{s(\tau s+1)}} = \frac{\mu}{s(\tau s+1)+P\mu(sT+1)} = \frac{\mu}{s^2\tau + s(P\mu T+1)+P\mu} \qquad 5.$$

The denominator is the same as $T(s)$ so we have the same poles, and no zeroes.

The static gain is: $S_i(0) = \frac{\mu}{P\mu} = \frac{1}{P}$





We can see that at "low" frequency the attenuation mainly comes from the proportional gain.

Assuming to have a "step": $\left(A \times \text{step}(t) \rightarrow \frac{A}{s}\right)$ as the disturbance, the final effect on the output is given by the final value theorem:

$$y_\infty^{\text{step}} = \lim_{s \to 0} s \frac{A}{s} S_i(s) = A S_i(0) = AP \tag{6.}$$

TODO:

- Check if it's consistent with the "fixed bias" experiments
- Write some conclusions: this feels like a "bad" disturbance rejection, is it a worthy trade-off for not using an integrator?